ECE 162B
Homework #1 Solutions
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Problem #1

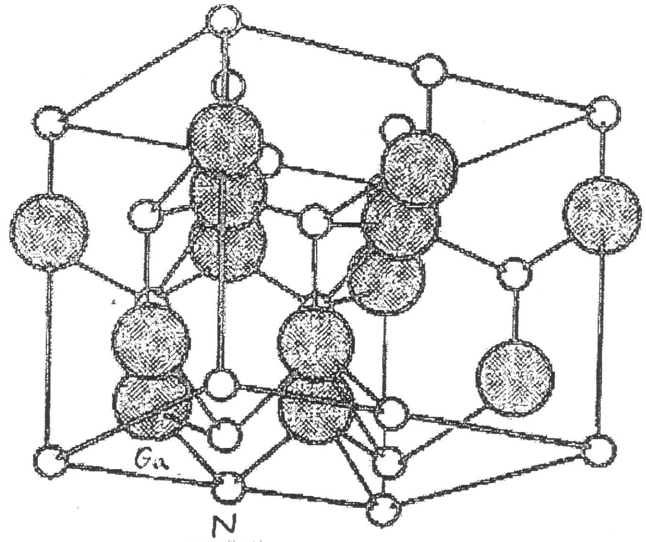
1.1) GaN crystallizes in Zinc-Blende and Wurtzite crystal structures. Zinc-Blende is a cubic crystal structure (see Kittel pg. 20). Wurtzite is a hexagonal crystal structure, and is the more common form of GaN. This problem concerns the Wurtzite structure, which follows:

Primitive Vectors:

$$\overline{A}_1 = \tfrac{1}{2}a\hat{x} - \tfrac{1}{2}\sqrt{3}\,a\hat{y}$$
$$\overline{A}_2 = \tfrac{1}{2}a\hat{x} + \tfrac{1}{2}\sqrt{3}\,a\hat{y}$$
$$\overline{C} = c\hat{z}$$

Basis:

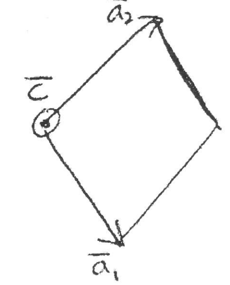
$$\overline{B}_1 = \tfrac{1}{3}\overline{A}_1 + \tfrac{2}{3}\overline{A}_2 \quad (Ga)$$
$$\overline{B}_2 = \tfrac{2}{3}\overline{A}_1 + \tfrac{1}{3}\overline{A}_2 + \tfrac{1}{2}\overline{C} \quad (Ga)$$
$$\overline{B}_3 = \tfrac{1}{3}\overline{A}_1 + \tfrac{2}{3}\overline{A}_2 + u\overline{C} \quad (N)$$
$$\overline{B}_4 = \tfrac{2}{3}\overline{A}_1 + \tfrac{1}{3}A_2 + (u+\tfrac{1}{2})\overline{C} \quad (N)$$



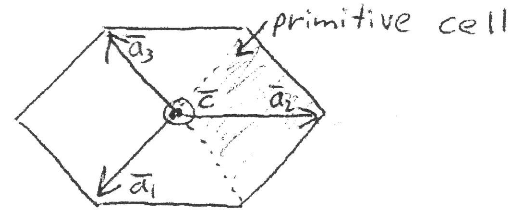
(adapted from cst-www.nrl.navy.mil/lattice/struk/b4.html)

The conventional cell, a hexagon, can be formed by tiling primitive cells. The Wurtzite basis for the primitive cell consists of two gallium and two nitrogen atoms located at the positions given by the basis vectors. Note that none of the basis vectors place atoms on lattice points (i.e. points that form the conventional hexagonal cell). This makes the structure difficult to visualize. The parameter u is dependent on the relative sizes of the two elements in the Wurtzite structure. Let us approximate GaN as an ideal Wurtzite structure, in which the two elements are of unequal size and as closely packed as possible. In this case, $\frac{c}{a} = \sqrt{\frac{8}{3}}$ as in hcp and $u = \frac{3}{8}$. A sketch of the primitive cell follows. Note that the primitive cell is only $\frac{1}{3}$ of the conventional hexagonal cell.

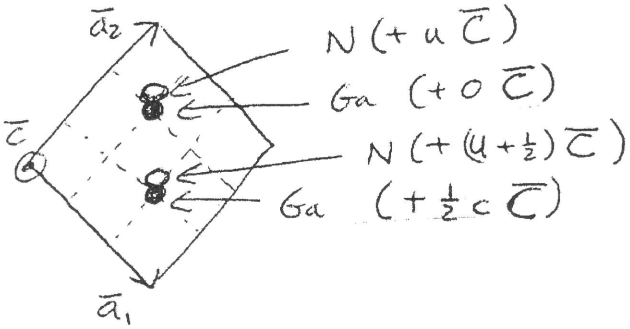
Primitive Cell:



Conventional (Hexagonal) Cell:

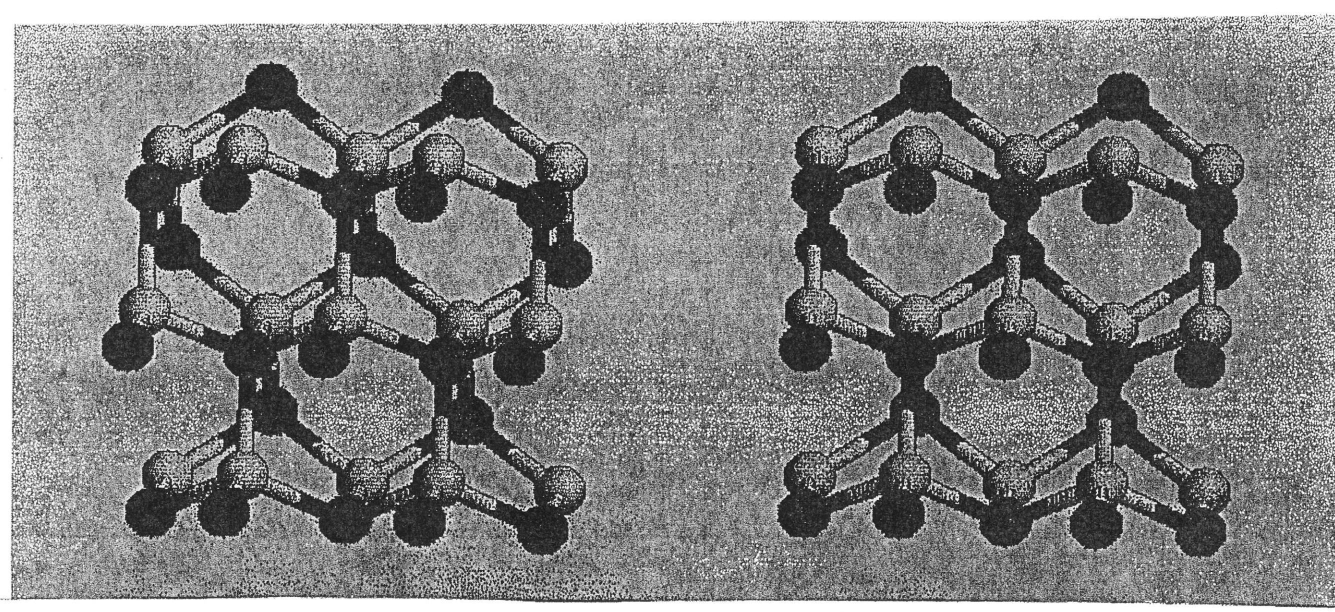


Primitive Cell with Wurtzite Basis:



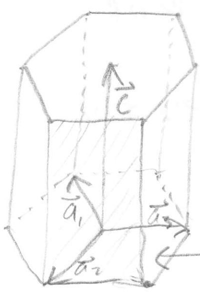
This is a projection in the (001) plane.

The following is a stereoscopic model of the Wurtzite structure. Cross your eyes and focus. You should be able to see the structure in 3D. Perhaps the best ay to describe Wurtzite is as a structure in which the element with a larger radius (Ga) forms an hcp structure, with the smaller radius element (N) filling half of the interstital voids.



The atoms are not shown at actual size in this model.

1.2 IN THE HEXAGONAL SYSTEM, THE (h,k,i,l) PLANE INTERCEPTS THE CRYSTAL AXES AT $\frac{1}{h}\vec{a}_1$, $\frac{1}{k}\vec{a}_2$, $\frac{1}{i}\vec{a}_3$ AND $\frac{1}{l}\vec{c}$, SO THE $(\bar{1},1,0,0)$ PLANE LOOKS LIKE THIS IN THE CONVENTIONAL HEXAGONAL CELL:



INTERCEPTS ARE: $\vec{a}_1$ AT -1
$\vec{a}_2$ AT 1
$\vec{a}_3$ AT ∞
$\vec{c}$ AT ∞

$(\bar{1},1,0,0)$ PLANE

I HAVEN'T DRAWN THE GA + N ATOMS BECAUSE THEY MAKE THE SKETCH OVERLY COMPLICATED. TO VISUALIZE THE ATOM LOCATIONS, IMAGINE TILING THE PRIMITIVE CELLS:

CONVENTIONAL (HEXAGONAL) CELL

$(\bar{1},1,0,0)$ PLANE

●: GA
○: N

PRIMITIVE CELLS

THE ABOVE IS A PROJECTION IN THE $(0,0,0,1)$ PLANE. REMEMBER THE GA + N ATOMS ARE LOCATED AT DISTANCES ABOVE THE PLANE ACCORDING TO THE BASIS VECTORS.

1.3) THE DENSITY OF GAN IS THE RATIO OF THE MASS OF THE 2 GA ATOMS & 2 N ATOMS IN A PRIMITIVE CELL TO THE VOLUME OF THE PRIMITIVE CELL.

$$\text{DENSITY} = \frac{\text{MASS}}{\text{VOLUME}}$$

MASSES

GA → 69.7 g/mol

N → 14.0 g/mol

VOLUME OF PRIMITIVE CELL (PARALLELEPIPED)

$$\rightarrow V = \vec{a}_1 \cdot (\vec{a}_2 \times \vec{c})$$

$$= \left(\frac{a}{2}\hat{x} - \frac{\sqrt{3}}{2}a\hat{y}\right) \cdot \left[\left(\frac{a}{2}\hat{x} + \frac{\sqrt{3}}{2}a\hat{y}\right) \times (c\hat{z})\right]$$

$$= \left(\frac{a}{2}\hat{x} - \frac{\sqrt{3}}{2}a\hat{y}\right) \cdot \left(-\frac{ac}{2}\hat{y} + \frac{\sqrt{3}}{2}ac\hat{x}\right) = \frac{\sqrt{3}}{4}a^2c + \frac{\sqrt{3}}{4}a^2c = \underline{\frac{\sqrt{3}}{2}a^2c}$$

AT 300K IN GAN, $a = 3.19$ Å & $c = 5.185$ Å

NOTICE $c \approx \sqrt{\frac{8}{3}}\,a$, THE RATIO FOR CLOSE PACKING IN HCP.

$$\text{SO DENSITY} = \frac{[2 \cdot 69.7\ \text{g/mol} + 2 \cdot 14.0\ \text{g/mol}]\,[\text{mol}/6.022 \cdot 10^{23}]}{\frac{\sqrt{3}}{2} \cdot (3.19 \cdot 10^{-8}\ \text{cm})^2 \cdot (5.185 \cdot 10^{-8}\ \text{cm})}$$

$$= \boxed{6.08\ \text{g/cm}^3}$$

Problem #~~3~~ 2

HCP has the following lattice and basis vectors:

- Lattice Vectors (Hexagonal Lattice Vectors):

$\bar{A}_1 = -\frac{1}{2}a\hat{x} - \frac{1}{2}\sqrt{3}\,a\hat{y}$

$\bar{A}_2 = a\hat{x}$ (The redundant vector $\bar{A}_3$ has been omitted.)

$\bar{C} = c\hat{z}$

Basis:

$\bar{B}_1 = 0$ (on each lattice point)

$\bar{B}_2 = \frac{2}{3}\bar{A}_1 + \frac{1}{3}\bar{A}_2 + \frac{1}{2}\bar{C}$

In HCP, all atoms are separated by the same distance, a, the lattice constant. Therefore, the distance between the basis vectors must be a:

$$d = d_{\bar{B}_2 - \bar{B}_1} = \left|\tfrac{2}{3}\bar{A}_1 + \tfrac{1}{3}\bar{A}_2 + \tfrac{1}{2}\bar{C}\right|$$

$$d = \left|\tfrac{2}{3}\left(-\tfrac{1}{2}a\hat{x} - \tfrac{1}{2}\sqrt{3}\,a\hat{y}\right) + \tfrac{1}{3}(a\hat{x}) + \tfrac{1}{2}(c\hat{z})\right|$$

$$d = \left|-\tfrac{1}{3}a\hat{x} - \tfrac{a}{\sqrt{3}}\hat{y} + \tfrac{1}{3}a\hat{x} + \tfrac{1}{2}c\hat{z}\right|$$

$$d = \left|-\tfrac{a}{\sqrt{3}}\hat{y} + \tfrac{1}{2}c\hat{z}\right|$$

$$d = \sqrt{\left(\tfrac{a}{\sqrt{3}}\right)^2 + \left(\tfrac{c}{2}\right)^2}$$

Since $d = a$:

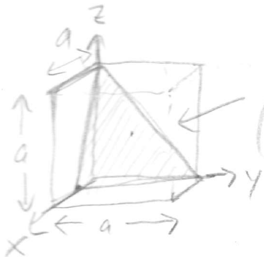
$$a^2 = d^2 = \frac{a^2}{3} + \frac{c^2}{4}$$

$$\frac{2}{3}a^2 = \frac{1}{4}c^2$$

$$\boxed{\frac{c}{a} = \sqrt{\frac{8}{3}}}$$

Many other geometrical methods can be used to achieve this result.

3.1



$\leftarrow$ (3, 1, 1) PLANE

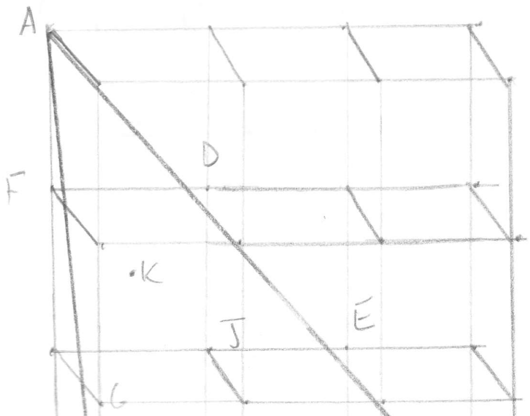
INTERCEPTS AT: $\frac{1}{3}a\hat{x}$, $a\hat{y}$, $a\hat{z}$

LATTICE TRANSLATION VECTORS: $\vec{a}_1 = a\hat{x}$
$\vec{a}_2 = a\hat{y}$
$\vec{a}_3 = a\hat{z}$

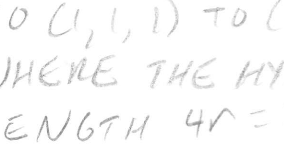
BASIS VECTORS: $\vec{B}_1 = 0$
$\vec{B}_2 = \frac{1}{2}(\vec{a}_1 + \vec{a}_2 + \vec{a}_3)$

$$\text{ATOMIC DENSITY} = \frac{\#\text{ ATOMS}}{\text{VOLUME}} = \frac{2}{(2 \cdot 10^{-8}\text{ cm})^3} = 2.5 \cdot 10^{23}\ \frac{\text{ATOMS}}{\text{cm}^3}$$

3.2 SINCE THE INTERSECTION OF THE (3, 1, 1) PLANE HAS A DIFFERENT DENSITY IN ADJACENT CONVENTIONAL CELLS, CONSIDER A PORTION OF THE (3, 1, 1) PLANE WHICH CAN BE TILED TO PRODUCE THE ENTIRE PLANE, AND THEREFORE HAS THE SAME ATOMIC DENSITY AS THE ENTIRE PLANE, SUCH AS THE TRIANGLE BELOW WITH INTERCEPTS AT (1, 0, 0), (0, 3, 0) & (0, 0, 3)



ASSUMING ELEMENT X TO BE CLOSE-PACKED, ONE CAN DRAW A TRIANGLE FROM (0, 0, 0) TO (1, 1, 0) TO (1, 1, 1) TO (0, 0, 0) WHERE THE HYPOTENUSE HAS LENGTH $4r = \sqrt{3}a$ SINCE THE CRYSTAL IS CLOSE PACKED:



THE EQUATION OF PLANE IS $\hat{n}\cdot(\vec{r}-\vec{r}_0)=0$

WHERE $\hat{n}$ IS THE UNIT NORMAL $\dfrac{3\hat{x}+\hat{y}+\hat{z}}{\sqrt{11}}$

$\vec{r}_0$ IS A POINT IN THE PLANE, LIKE $\hat{x}$, THEN

$$(3\hat{x}+\hat{y}+\hat{z})\cdot(x\hat{x}+y\hat{y}+z\hat{z}-\hat{x}) = 3(x-1)+y+z=0$$

①

$$\Rightarrow \underline{3x+y+z=3}$$

ALSO, THE EQUATIONS OF A LINE PASSING THROUGH

$\vec{r}_0$ IS $\hat{n}\times(\vec{r}-\vec{r}_0)=0$, THEN $(3\hat{x}+\hat{y}+\hat{z})\times[(x-x_0)\hat{x}+(y-y_0)\hat{y}+(z-z_0)\hat{z}]$

② ③

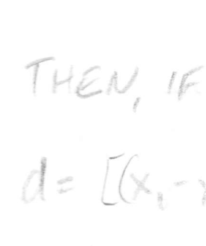
$$\Rightarrow 3(y-y_0)=(x-x_0)=3(z-z_0)$$

USING THESE EQUATIONS, YOU CAN VERIFY THAT TRIANGLE ABC INTERSECTS THE 13 ATOMS LABELED A THROUGH M SINCE A LINE PASSING THROUGH THE CENTER OF AN ATOM AT $\vec{r}_0$ AND NORMAL TO THE (3,1,1) PLANE WILL INTERSECT THE (3,1,1) PLANE AT A POINT SATISFYING EACH OF EQUATIONS ①, ② + ③, SAY POINT $\vec{r}_1=(x_1, y_1, z_1)$.

THEN, IF THE DISTANCE BETWEEN THE ATOM AND PLANE (3,1,1),

$$d=[(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2]^{1/2} < r=\sqrt{\frac{3}{16}}\,a$$

THE TRIANGLE ABC INTERSECTS THE ATOM. THE RADIUS OF THE CIRCLE WHERE ABC INTERSECTS THE ATOM IS EVIDENTLY



(3,1,1) PLANE $\underline{a=(r^2-d^2)^{1/2}}$

THEN, THE ATOMIC DENSITY OF PLANE (3, 1, 1), D, IN ATOMS / CM^2 IS THE SUM OF EACH OF THE INTERSECTING CIRCLES A - M, DIVIDED BY THE CROSS-SECTIONAL AREA OF AN ATOM, πr^2, DIVIDED BY THE AREA OF TRIANGLE ABC.

BEGINNING WITH POINTS A, B, C WHICH ARE THE VERTICES OF THE TRIANGLE ABC AND THE CENTERS OF ATOMS, WE SEE THAT THE AREA OF INTERSECTION SATISFIES $\left(\frac{\angle A}{360°}\cdot\pi r^2\right)+\left(\frac{\angle B}{360°}\cdot\pi r^2\right)+\left(\frac{\angle C}{360°}\cdot\pi r^2\right)$

$$= \left(\frac{\angle A+\angle B+\angle C}{360°}\right)\cdot\pi r^2 = \frac{180°}{360°}\cdot\pi r^2 = \frac{\pi r^2}{2}$$

SINCE THE SUM OF INTERIOR ANGLES OF A TRIANGLE IS 180°.

SINCE EDGE $\overline{AB}$ PASSES THROUGH D + E, THE ATOMS AT THESE POINTS EACH CONTRIBUTE

$$\frac{180°}{360°}\cdot\pi r^2 = \frac{\pi r^2}{2}$$

PLANE (3, 1, 1) INTERSECTS ATOMS F + G, BUT EDGE $\overrightarrow{AC}$ PASSES THROUGH BOTH CIRCLES OF INTERSECTION, SO ATOMS F + G EACH CONTRIBUTE ONLY THE AREA OF THEIR INTERSECTING CIRCLES TO THE RIGHT OF $\overline{AC}$, WHERE $y > 0$.

THE POINT ON PLANE (3, 1, 1) CLOSEST TO ATOM F, CENTERED AT (0, 0, 2), WILL SATISFY ①, ② + ③:

$$3x + y + z = 3, \quad 3y = x = 3(z-2) = 3z - 6$$

$$\Rightarrow (x, y, z) = \left(\frac{3}{11}, \frac{1}{11}, \frac{23}{11}\right) \Rightarrow d_F = \left[\left(\frac{3}{11}\right)^2 + \left(\frac{1}{11}\right)^2 + \left(\frac{1}{11}\right)^2\right]^{1/2} = \frac{1}{\sqrt{11}} = d_F$$

THE NORMAL TO EDGE $\overline{AC}$ PASSING THROUGH $\left(\frac{3}{11}, \frac{1}{11}, \frac{23}{11}\right)$

IS GIVEN BY $\overline{AC} \cdot \left(\vec{r} - \left(\frac{3}{11}, \frac{1}{11}, \frac{23}{11}\right)\right) = 0$

$$\Rightarrow (3\hat{z} - \hat{x}) \cdot \left[\left(x - \frac{3}{11}\right)\hat{x} + \left(y - \frac{1}{11}\right)\hat{y} + \left(z - \frac{23}{11}\right)\hat{z}\right] = 3z - \frac{69}{11} - x + \frac{3}{11} = 0$$

$\Rightarrow 3z - x = 6$ ALSO $3x + y + z = 3$ SINCE THIS LINE IS IN $(3,1,1)$

AND $y = 0$ SINCE WE'RE FINDING THE POINT ON $\overline{AC}$ CLOSEST TO $\left(\frac{3}{11}, \frac{1}{11}, \frac{23}{11}\right)$

$$\Rightarrow (x, y, z) = \left(\frac{3}{10}, 0, \frac{21}{10}\right) \Rightarrow v_F = \left[\left(\frac{3}{10} - \frac{3}{11}\right)^2 + \left(\frac{1}{11} - 0\right)^2 + \left(\frac{21}{10} - \frac{23}{11}\right)^2\right]^{1/2} =$$

$$\left[\left(\frac{3}{110}\right)^2 + \left(\frac{10}{110}\right)^2 + \left(\frac{1}{110}\right)^2\right]^{1/2} = \frac{1}{\sqrt{110}} = v_F$$

$(1,0,1)$

ALSO, FOR ATOM G, THE CLOSEST POINT ON PLANE $(3,1,1)$ IS GIVEN BY

$$3x + y + z = 3, \quad 3y = x - 1 = 3(z-1) \Rightarrow (x, y, z) = \left(\frac{8}{11}, -\frac{1}{11}, \frac{10}{11}\right)$$

$$\Rightarrow d_G = \left[\left(\frac{11}{11} - \frac{8}{11}\right)^2 + \left(0 - \frac{-1}{11}\right)^2 + \left(\frac{11}{11} - \frac{10}{11}\right)^2\right]^{1/2} = \frac{1}{\sqrt{11}} = \underline{d_G = d_F}$$

AND THE CLOSEST POINT ON $\overline{AC}$ TO $\left(\frac{8}{11}, \frac{-1}{11}, \frac{10}{11}\right)$ IS GIVEN BY

$$3x + y + z = 3, \quad (3\hat{z} - \hat{x}) \cdot \left[\left(x - \frac{8}{11}\right)\hat{x} + \left(y + \frac{1}{11}\right)\hat{y} + \left(z - \frac{10}{11}\right)\hat{z}\right] = 3z - \frac{30}{11} - x + \frac{8}{11} = 0$$

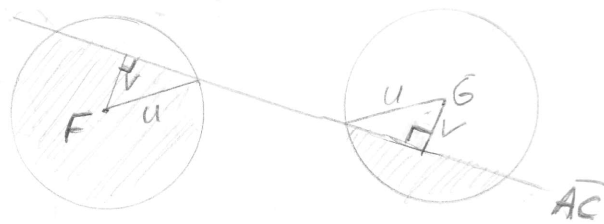
$$\Rightarrow 3z - x = 2 \quad \& \quad y = 0, \text{ so } (x, y, z) = \left(\frac{7}{10}, 0, \frac{9}{10}\right) \Rightarrow v_G = \left[\left(\frac{7}{10} - \frac{8}{11}\right)^2 + \left(0 + \frac{1}{11}\right)^2 + \left(\frac{9}{10} - \frac{10}{11}\right)^2\right]^{1/2}$$

$$v_G = \left[\left(\frac{3}{110}\right)^2 + \left(\frac{10}{110}\right)^2 + \left(\frac{1}{110}\right)^2\right] = \frac{1}{\sqrt{110}} = \underline{v_G = v_F}$$

SO THE AREA OF INTERSECTION OF TRIANGLE ABC WITH ATOM F IS A TRUNCATED CIRCLE OF RADIUS $u = (r^2 - d_F^2)^{1/2}$ IN WHICH THE TRUNCATING LINE, $\overline{AC}$, PASSES THE CENTER, F,

AT A DISTANCE v_F. SIMILARLY, THE AREA OF INTERSECTION OF TRIANGLE ABC WITH ATOM G IS A TRUNCATED CIRCLE OF THE SAME DIMENSIONS, $d_G = d_F$ + $v_G = v_F$. HOWEVER, THE CENTERS OF THESE TWO IDENTICAL CIRCLES ARE ON EITHER SIDE OF $\overline{AC}$ SINCE $y = \frac{1}{11}$ FOR F AND $y = \frac{-1}{11}$ FOR G, SO THE TWO AREAS ARE COMPLEMENTARY AND ATOMS F AND G CONTRIBUTE AREA $\pi u^2 = \pi(r^2 - d_F^{\,2})$

$$= \pi\left(r^2 - \frac{a^2}{11} \cdot \frac{16 r^2}{3a^2}\right) = \frac{17\pi r^2}{33}$$



FOR ATOM H, $(1,1,0)$ WE HAVE

$$3x + y + z = 3, \quad 3(y-1) = (x-1) = 3(z-0)$$

$$\Rightarrow (x,y,z) = \left(\frac{8}{11}, \frac{10}{11}, \frac{-1}{11}\right) \Rightarrow d_H = \frac{1}{\sqrt{11}},\ v_H = \frac{1}{\sqrt{110}}$$

FOR ATOM I, $(0,2,0)$ WE HAVE

$$3x + y + z = 3 \quad 3(y-2) = x - 0 = 3(z-0)$$

$$\Rightarrow (x,y,z) = \left(\frac{3}{11}, \frac{23}{11}, \frac{1}{11}\right) \Rightarrow d_I = \frac{1}{\sqrt{11}},\ v_I = \frac{1}{\sqrt{110}}$$

AND AGAIN THE TWO CIRCLES OF INTERSECTION ARE CENTERED ON OPPOSITE SIDES OF THE TRUNCATING LINE $\overline{CB}$, SO THEIR AREAS ARE EQUAL TO THOSE OF ATOMS F + G: $\frac{17\pi r^2}{33}$

ATOM J IS SITUATED RELATIVE TO PLANE (3, 1, 1) IN THE SAME WAY AS ATOM F, EXCEPT ITS CIRCLE OF INTERSECTION WITH PLANE (3, 1, 1)

IS NOT TRUNCATED BY ANY EDGE OF TRIANGLE ABC, SO ATOM J CONTRIBUTES AREA $\dfrac{17\pi r^2}{33}$

FOR ATOM K, $\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right)$

$$3x + y + z = 3 \qquad 3\left(y - \tfrac{1}{2}\right) = x - \tfrac{1}{2} = 3\left(z - \tfrac{3}{2}\right)$$

$$3y - 1 = x = 3z - 4$$

$$\Rightarrow (x, y, z) = \left(\tfrac{4}{11}, \tfrac{5}{11}, \tfrac{16}{11}\right) \Rightarrow d_K^2 = \left(\tfrac{4}{11} - \tfrac{1}{2}\right)^2 + \left(\tfrac{5}{11} - \tfrac{1}{2}\right)^2 + \left(\tfrac{16}{11} - \tfrac{3}{2}\right)^2$$

$$d_K = \left[\left(\tfrac{3}{22}\right)^2 + \left(\tfrac{1}{22}\right)^2 + \left(\tfrac{1}{22}\right)^2\right]^{1/2} = \underline{\frac{1}{2\sqrt{11}} = d_K}$$

SO ATOM K CONTRIBUTES $\pi\left(r^2 - \left(\frac{\cancel{a}}{2\sqrt{11}}\right)^2 \cdot \frac{16r^2}{3\cancel{a^2}}\right) = \dfrac{29\pi r^2}{33}$

FOR ATOM L, $\left(\frac{1}{2}, \frac{3}{2}, \frac{1}{2}\right)$

$$3x + y + z = 3 \qquad 3\left(y - \tfrac{3}{2}\right) = x - \tfrac{1}{2} = 3\left(z - \tfrac{1}{2}\right)$$

$$\Rightarrow (x, y, z) = \left(\tfrac{4}{11}, \tfrac{16}{11}, \tfrac{5}{11}\right) \Rightarrow d_L = \frac{1}{2\sqrt{11}}$$

SO ATOM L CONTRIBUTES $\dfrac{29\pi r^2}{33}$

FOR ATOM M, $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

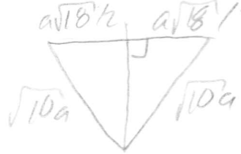
$$3x + y + z = 3 \qquad 3\left(y - \tfrac{1}{2}\right) = x - \tfrac{1}{2} = 3\left(z - \tfrac{1}{2}\right)$$

$$\Rightarrow (x, y, z) = \left(\tfrac{7}{11}, \tfrac{6}{11}, \tfrac{6}{11}\right) \Rightarrow d_M = \left[\left(\tfrac{7}{11} - \tfrac{1}{2}\right)^2 + \left(\tfrac{6}{11} - \tfrac{1}{2}\right)^2 + \left(\tfrac{6}{11} - \tfrac{1}{2}\right)^2\right]^{1/2} = \frac{1}{2\sqrt{11}}$$

SO ATOM M CONTRIBUTES $\dfrac{29\pi r^2}{33}$

TRIANGLE ABC IS ISOSCELES, WITH EDGES

$\sqrt{10}a, \sqrt{10}a, \sqrt{18}a$



& HEIGHT $\left(10a^2 - \frac{18a^2}{4}\right)^{1/2} = \sqrt{\frac{11}{2}}\,a$

SO ITS AREA IS $\frac{a\sqrt{18}}{2} \cdot a\frac{\sqrt{11}}{\sqrt{2}} = \frac{3\sqrt{11}}{2}a^2$

THE TOTAL ATOMIC DENSITY OF TRIANGLE ABC, AND BY EXTENSION, PLANE (3, 1, 1), IS

$$\cancel{\pi r^2}\Big[\overset{A,B,C}{1/2} + \overset{D}{\frac{1}{2}} + \overset{E}{\frac{1}{2}} + \overset{F,G}{\frac{17}{33}} + \overset{H,I}{\frac{17}{33}} + \overset{J}{\frac{17}{33}}$$

$$+ \overset{K}{\frac{29}{33}} + \overset{L}{\frac{29}{33}} + \overset{M}{\frac{29}{33}}\Big] \cdot \frac{1\text{ ATOM}}{\cancel{\pi r^2}} \cdot \frac{2}{3\sqrt{11}a^2} = \frac{125}{33\sqrt{11}(2\cdot 10^{-8}\text{ cm})^2}$$

$$= 2.86\cdot 10^{15}\text{ ATOMS/cm}^2$$

NOTICE THE DENSITY OF THE (1, 0, 0) PLANE IS $\frac{1\text{ ATOM}}{(2\cdot10^{-8}\text{ cm})^2} = 2.5\cdot 10^{15}\,\frac{\text{ATOMS}}{\text{cm}^2}$